Arizona Mathematics Standard Articulated by Grade Level

GRADE 6

Every student should understand and use all concepts and skills from the previous grade levels. The standard is designed so that new learning builds on preceding skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of all mathematical strands.

Strand 1: Number and Operations

Number sense is the understanding of numbers and how they relate to each other and how they are used in specific context or real-world application. It includes an awareness of the different ways in which numbers are used, such as counting, measuring, labeling, and locating. It includes an awareness of the different types of numbers such as, whole numbers, integers, fractions, and decimals and the relationships between them and when each is most useful. Number sense includes an understanding of the size of numbers, so that students should be able to recognize that the volume of their room is closer to 1,000 than 10,000 cubic feet. Students develop a sense of what numbers are, i.e., to use numbers and number relationships to acquire basic facts, to solve a wide variety of real-world problems, and to estimate to determine the reasonableness of results.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, and the relationships among numbers and different number systems.

In Grade 6, students broaden their knowledge of fractions, decimals, percents, and ratios, and the relationships between each. They compare and order integers, fractions, decimals, and percents. They explore the inverse relationships between perfect squares and cubes, and their roots and are introduced to absolute value.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Convert between expressions for positive rational numbers, including fractions, decimals, percents, and ratios. Connections: M06-S1C1-03, M06-S1C1-04, M06-S1C3-01, M06-S2C2-01, M06-S2C2-02	M06-S5C2-05. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	Students need many opportunities to use multiple representations in meaningful contexts. Continued on next page

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
		Example: • A baseball player's batting average is 0.625. What is his batting average when written as a fraction, ratio, and percent? Solution: o The player hit the ball $\frac{5}{8}$ of the time they were at bat; o The player hit the ball 62.5% of the time; or o The player has a ratio of 5 hits to 8 at bats (5:8).
PO 2. Use prime factorization to • express a whole number as a product of its prime factors and • determine the greatest common factor and least common multiple of two whole numbers. Connections: M06-S1C1-06	M06-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	Students are expected to use exponents where appropriate to summarize the prime factors. Examples: • What is the prime factorization of 24? (solution: $2^3 \cdot 3$) • What is the prime factorization of 36? (solution: $2^2 \cdot 3^2$) • What is the greatest common factor (GCF) of 24 and 36? How can you use the prime factorizations to find the GCF? (solution: $2^2 * 3 = 12$) • What is the least common multiple (LCM) of 24 and 36? How can you use the prime factorizations to find the LCM? (solution: $2^3 * 3^2 = 72$)
PO 3. Demonstrate an understanding of fractions as rates, division of whole numbers, parts of a whole, parts of a set, and locations on a real number line.	M06-S5C2-05. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	Students are expected to demonstrate understanding when working with fractions in multiple contexts. These contexts include but are not limited to common rates (charges/minutes, cost/item, miles/gallon, miles/hour), fair share problems, ratio tables, number lines, and expressions. This builds on students' previous work with ratios and unit rates in grade 5. Continued on next page

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
Connections: M06-S1C1-01, M06-S1C1-04, M06-S4C4-02, M06-S4C4-03	M06-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	Examples: • The Gab Line Phone Company charges $20.00/month plus $0.05/minute for cell phone service, and $0.10/text message. If you used 246 minutes and sent 454 text messages, how much should you expect your bill this month to be? Does this fall within the $50 limit your parents have set? • Students should recognize the fraction bar as a grouping symbol that indicates division in the context of expressions. $\frac{3(2+0.5)}{7}$ can also be written as [3(2+0.5)]÷7. • Two afterschool clubs are having pizza parties. For the Math Club, the teacher will order 3 pizzas for every 5 students. For the student council, the teacher will order 5 pizzas for every 8 students. Since you are in both groups, you need to decide which party to attend. How much pizza would you get at each party? If you want to have the most pizza, which party should you attend? • The science club is donating a fish tank for the front office. They want to make a replica of the fish tank in their classroom but 4 times larger. There are 40 fish in the classroom tank, with a ratio of 5:3 (goldfish to guppies). How many of each type of fish will be needed for the larger tank? Write your answer as a ratio that describes the number of each type of fish. • Draw a number line to show the placement of $2\frac{4}{5}$. Continued on next page

<table>
<tr><th>Performance Objectives</th><th>Process Integration</th><th>Explanations and Examples</th></tr>
<tr><td>Students are expected to:</td><td></td><td></td></tr>
<tr><td></td><td></td><td>• A credit card company charges 17% interest on any charges not paid at the end of the month. Make a ratio table to show how much the interest would be for several amounts. If your bill totals $450 for this month, how much interest would you have to pay if you let the balance carry to the next month?

<table><tr><td>Charges</td><td>$1</td><td>$50</td><td>$100</td><td>$200</td><td>$450</td></tr><tr><td>Interest</td><td>$0.17</td><td>$8.50</td><td>$17</td><td>$34</td><td>?</td></tr></table></td></tr>
<tr><td>PO 4. Compare and order integers; and positive fractions, decimals, and percents.

Connections: M06-S1C1-01, M06-S1C1-03, M06-S1C3-01, M06-S1C3-02</td><td>M06-S5C2-03. Analyze and compare mathematical strategies for efficient problem solving; select and use one or more strategies to solve a problem.</td><td>Positive rational numbers include values greater than zero, such as proper fractions, improper fractions, mixed numbers, and percents both greater and less than 100%.

Example:
• List the numbers: $\frac{2}{3}, -3, 1.2, -1, \frac{11}{5}$ in increasing order. Explain the strategies you used to order the numbers.</td></tr>
<tr><td>PO 5. Express that a number’s distance from zero on the number line is its absolute value.

Connections: M06-S1C2-01</td><td></td><td>Content critical to development of student understanding of absolute value include the definition of absolute value, a visual representation of absolute value on a number line, and the symbols used to designate absolute value.</td></tr>
<tr><td>PO 6. Express the inverse relationships between exponents and roots for perfect squares and cubes.

Connections: M06-S1C1-02</td><td>M06-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.</td><td>Examples:
• $2^2 = 2 \cdot 2 = 4$ and $\sqrt{4} = \sqrt{2 \bullet 2} = 2$
• $2^3 = 2 \cdot 2 \cdot 2 = 8$ and $\sqrt[3]{8} = \sqrt[3]{2 \bullet 2 \bullet 2} = 2$</td></tr>
</table>

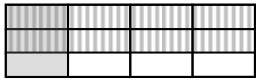
Strand 1: Number and Operations
Concept 2: Numerical Operations

Understand and apply numerical operations and their relationship to one another.

In Grade 6, students build upon their prior knowledge of operations with rational numbers by multiplying and dividing fractions and decimals. They extend their computation of decimals to include division of whole numbers and decimals by a decimal. They expand their understanding of the real number system by modeling the concepts of addition and subtraction of integers. Students simplify numerical expressions using order of operations that now include exponents. They continue to apply properties of the real number system to evaluate expressions.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Apply and interpret the concepts of addition and subtraction with integers using models. Connections: M06-S1C1-05	M06-S5C2-05. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	Students need multiple opportunities to build conceptual understanding of addition and subtraction of integers though models. Models may include, but are not limited to number lines, two color chips, and integer balances.
PO 2. Multiply multi-digit decimals through thousandths. Connections: M06-S1C2-05, M06-S1C2-06, M06-S1C2-07, M06-S1C3-02, M06-S3C1-01, M06-S3C3-04, M06-S5C1-01	M06-S5C2-04. Apply a previously used problem-solving strategy in a new context.	Students multiply with decimals efficiently and accurately as well as solve problems in both contextual and non-contextual situations.
PO 3. Divide multi-digit whole numbers and decimals by decimal divisors with and without remainders. Connections: M06-S1C2-05, M06-S1C2-06, M06-S1C2-07, M06-S1C3-02, M06-S3C1-01, M06-S3C3-04, M06-S5C1-01	M06-S5C2-04. Apply a previously used problem-solving strategy in a new context.	Students divide with decimals efficiently and accurately as well as solve problems in both contextual and non-contextual situations.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 4. Multiply and divide fractions. Connections: M06-S1C2-05, M06-S1C2-06, M06-S1C2-07, M06-S1C3-02, M06-S3C1-01, M06-S3C3-04, M06-S5C1-01	M06-S5C2-03. Analyze and compare mathematical strategies for efficient problem solving; select and use one or more strategies to solve a problem.	Students are expected to multiply and divide fractions including proper fractions, improper fractions and mixed numbers. They multiply and divide fractions efficiently and accurately as well as solve problems in both contextual and non-contextual situations.
PO 5. Provide a mathematical argument to explain operations with two or more fractions or decimals. Connections: M06-S1C2-02, M06-S1C2-03, M06-S1C2-04, M06-S1C2-07, M06-S5C1-01	M06-S5C2-08. Make and test conjectures based on information collected from explorations and experiments.	Mathematical arguments may include, but are not be limited to models, pictures, or written explanations that demonstrate conceptual understanding. Example: • The product of fractions can be demonstrated using an array model. o In the example $\frac{2}{3} \bullet \frac{1}{4}$, the first fraction (two thirds) is modeled by dividing the rectangle horizontally into 3 parts and then shading 2 of the 3 rectangles (shown by the gray shaded lines).  Continued on next page

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
		o The second fraction (one fourth) is modeled by dividing the rectangle vertically into 4 parts and then shading 1 of the 4 rectangles (shown by the dark shading). The product is modeled by the overlap of the shaded areas (there are 2 pieces in which the overlap is shaded and there are 12 pieces total  $\frac{2}{3} \bullet \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$
PO 6. Apply the commutative, associative, distributive, and identity properties to evaluate numerical expressions involving whole numbers. Connections: M06-S1C2-02, M06-S1C2-03, M06-S1C2-04, M06-S1C2-07	M06-S5C2-04. Apply a previously used problem-solving strategy in a new context.	Examples: • Which properties can you use to simplify this expression? Justify your choice. 4(3 + 2) • Simplify $6(20+4)$, with and without the use of the distributive property. • Evaluate $d-4(2d-5)+3e$ when d = 13 and e = 3. How can you use properties (commutative, associative and distributive) to help you evaluate the expression?

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 7. Simplify numerical expressions (involving fractions, decimals, and exponents) using the order of operations with or without grouping symbols. Connections: M06-S1C2-02, M06-S1C2-03, M06-S1C2-04, M06-S1C2-05, M06-S1C2-06	M06-S5C2-04. Apply a previously used problem-solving strategy in a new context.	Examples: • $4 \div \frac{1}{2} + 5^2$ • 7 + 0.25 (3.6 – 1.35)

Strand 1: Number and Operations
Concept 3: Estimation

Use estimation strategies reasonably and fluently while integrating content from each of the other strands.

In Grade 6, students continue to develop estimation strategies to predict and verify solutions. They use estimation to determine the reasonableness of solutions and continue to use benchmarks for the comparison of rational numbers.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Use benchmarks as meaningful points of comparison for rational numbers. Connections: M06-S1C1-01, M06-S1C1-04	M06-S5C2-05. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	Example: • Order the following numbers from least to greatest on a number line, and explain your reasoning. Which benchmarks were you able to use to help you order the numbers? 0.75, $\frac{1}{3}$, -2, $\sqrt{4}$

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 2. Make estimates appropriate to a given situation and verify the reasonableness of the results. Connections: M06-S1C1-04, M06-S1C2-02, M06-S1C2-03, M06-S1C2-04, M06-S2C1-03, M06-S2C2-02, M06-S3C3-02, M06-S3C3-04, M06-S3C4-01, M06-S4C4-01, M06-S4C4-02, M06-S4C4-03, M06-S4C4-04, M06-S4C4-05	M06-S5C2-01. Analyze a problem situation to determine the question(s) to be answered. M06-S5C2-02. Identify relevant, missing, and extraneous information related to the solution to a problem. M06-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Students should estimate using all four operations with whole numbers, fractions, and decimals. Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies for calculations with fractions and decimals extend from students' work with whole number operations. Estimation strategies include, but are not limited to: • front-end estimation with adjusting (using the highest place value and estimating from the front end making adjustments to the estimate by taking into account the remaining amounts), • clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate), • rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values), • using friendly or compatible numbers such as factors (students seek to fit numbers together - i.e., rounding to factors and grouping numbers together that have round sums like 100 or 1000), and • using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate). Specific strategies also exist for estimating measures. Students should develop fluency in estimating using standard referents (meters, yard, etc) or created referents (the window would fit about 12 times across the wall).

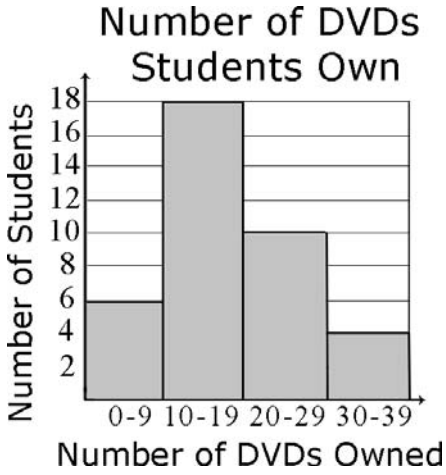
Strand 2: Data Analysis, Probability, and Discrete Mathematics
This strand requires students to use data collection, data analysis, statistics, probability, systematic listing and counting, and the study of graphs. This prepares students for the study of discrete functions as well as to make valid inferences, decisions, and arguments. Discrete mathematics is a branch of mathematics that is widely used in business and industry. Combinatorics is the mathematics of systematic counting. Vertex-edge graphs are used to model and solve problems involving paths, networks, and relationships among a finite number of objects.

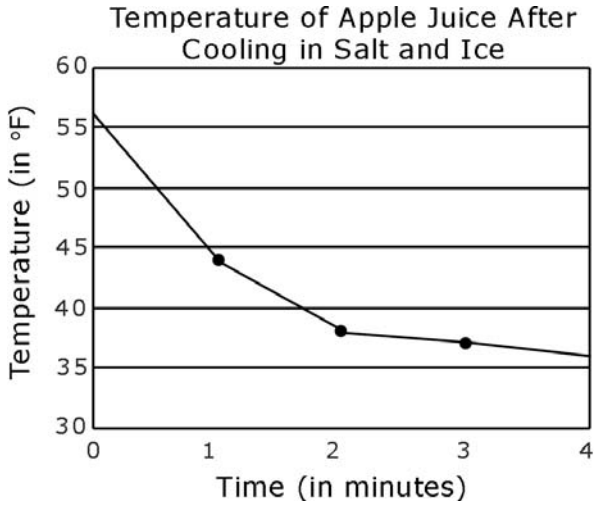
Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization, and representation to analyze and sort data.

In Grade 6, students apply their understanding of fractions, decimals, and percents as they construct, analyze, and describe data. They are introduced to data displays and summary statistics to analyze the distribution of data and compare two data sets.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Solve problems by selecting, constructing, and interpreting displays of data, including histograms and stem-and-leaf plots. Connections: M06-S2C1-02, M06-S2C1-03, M06-S2C1-04, SC06-S1C3-01, SC06-S1C3-04, SC06-S1C4-01, SC06-S1C4-02, SS06-S1C1-01, SS06-S1C1-02, SS06-S2C1-01, SS06-S2C1-02, SS06-S4C1-01, SS06-S4C1-02	M06-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	Students are expected to use appropriate labels, intervals, and title for an appropriate visual representation of collected data. Students will use histograms and stem-and-leaf plots in addition to all previously learned graphs. It is important that students have opportunities to choose the appropriate display for the representation of collected data.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 2. Formulate and answer questions by interpreting, analyzing, and drawing inferences from displays of data, including histograms and stem-and-leaf plots. Connections: M06-S2C1-01, M06-S2C1-03, M06-S2C1-04, SC06-S1C1-02, SC06-S1C3-04, SC06-S1C3-06, SS06-S1C1-02, SS06-S2C1-02, SS06-S4C1-02	M06-S5C2-01. Analyze a problem situation to determine the question(s) to be answered. M06-S5C2-02. Identify relevant, missing, and extraneous information related to the solution to a problem. M06-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language. M06-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Students are expected to make estimates and compute with a data set. Examples: • The histogram below shows the number of DVDs students own: o How many students own 20 or more DVDs? o How many students own fewer than 30 DVDs? o How many students own exactly 15 DVDs? (Students should notice that histograms display intervals, not individual pieces of data.)  Continued on next page

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
		• The line graph below shows the temperature of a can of juice over time, after placing it in an ice and salt mixture. Describe any conclusions you can make about the data. What are some possible questions you could ask using the data? 

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 3. Use extreme values, mean, median, mode, and range to analyze and describe the distribution of a given data set. Connections: M06-S1C3-02, M06-S2C1-01, M06-S2C1-02, M06-S2C1-04	M06-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Students use sets of data and graphical representations of data sets from real-world contexts. Example: • Use the stem and leaf plot below to determine the extreme values (maximum and minimum values represented), mean, median, mode and range. What do these values show about the distribution of the data? **Key: 2 | 3 = 23** **Spelling Test Scores** 1 | 3 2 | 3 | 4 | 5 | 2 4 6 | 3 3 7 | 4 5 6 6 7 8 | 2 5 7 8 9 9 | 0 1 3 4 4 10 | 0 0 0
PO 4. Compare two or more sets of data by identifying trends. Connections: M06-S2C1-01, M06-S2C1-02, M06-S2C1-03, SC06-S1C3-01	M06-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Students analyze data to identify trends (increasing, decreasing, constant). Students also analyze two or more sets of data to determine how the trends in multiple sets of data compare.

Strand 2: Data Analysis, Probability, and Discrete Mathematics
Concept 2: Probability

Understand and apply the basic concepts of probability.

In Grade 6, students begin to make and test conjectures about theoretical probability by predicting outcomes of experiments, performing experiments, comparing experimental outcomes to a prediction, and replicating experiments for the comparison of results. They determine possible outcomes using a variety of systematic approaches.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Use data collected from multiple trials of a single event to form a conjecture about the theoretical probability. Connections: M06-S1C1-01, M06-S2C2-02, M06-S2C2-03	M06-S5C2-08. Make and test conjectures based on information collected from explorations and experiments.	Example: • Each group receives a bag that contains 4 green marbles, 6 red marbles, and 10 blue marbles. Each group performs 50 pulls, recording the color of marble drawn and replacing the marble into the bag before the next draw. Students compile their data as a group and then as a class. They summarize their data as experimental probabilities and make conjectures about theoretical probabilities (How many green draws would you expect if you were to conduct 1000 pulls? 10,000 pulls?). Students create another scenario with a different ratio of marbles in the bag and make a conjecture about the outcome of 50 marble pulls with replacement. (An example would be 3 green marbles, 6 blue marbles, 3 blue marbles.) Students try the experiment and compare their predictions to the experimental outcomes to continue to explore and refine conjectures about theoretical probability.

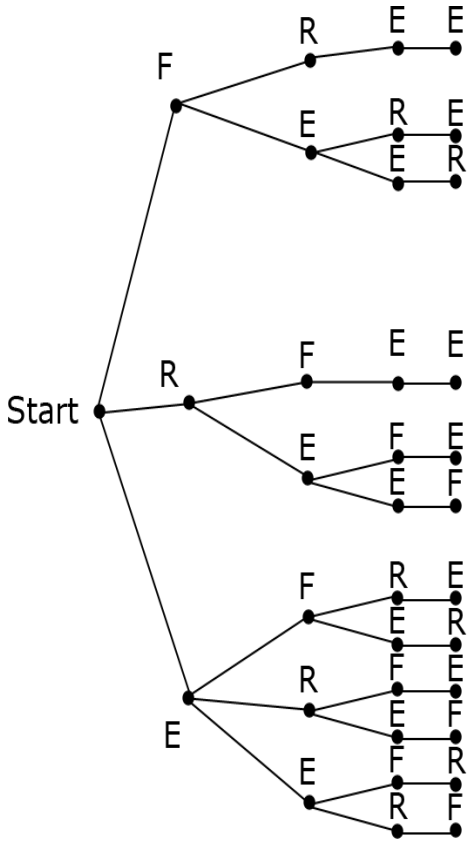
Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 2. Use theoretical probability to • predict experimental outcomes, • compare the outcome of the experiment to the prediction, and • replicate the experiment and compare results. Connections: M06-S1C1-01, M06-S1C3-02, M06-S2C2-01, M06-S2C2-03	M06-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Students need multiple opportunities to perform probability experiments and compare these results to theoretical probabilities. Critical components of the experiment process are making predictions about the outcomes by applying the principles of theoretical probability, comparing the predictions to the outcomes of the experiments, and replicating the experiment to compare results. Experiments can be replicated by the same group or by compiling class data. Experiments can be conducted using various random generation devices including, but not limited to, bag pulls, spinners, number cubes, coin toss, and colored chips.
PO 3. Determine all possible outcomes (sample space) of a given situation using a systematic approach. Connections: M06-S2C2-01, M06-S2C2-02, M06-S2C3-01	M06-S5C2-05. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	Systematic approaches may include, but are not limited to, frequency tables, tree diagrams, charts/tables, ordered pairs, and matrices. Example: • What are all of the outcomes of flipping a coin three times? <u>Systematic List</u> HHH TTT HTH HHT THH HTT TTH THT

Strand 2: Data Analysis, Probability, and Discrete Mathematics
Concept 3: Systematic Listing and Counting

Understand and demonstrate the systematic listing and counting of possible outcomes.

In Grade 6, students explore three attribute counting problems using Venn diagrams to build on prior learning about different counting problems. They learn to create and analyze tree diagrams where data repeats and expand their prior learning of the multiplication principle of counting.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Build and explore tree diagrams where items repeat. Connections: M06-S2C2-03	M06-S5C2-05. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	Students have had opportunities to build tree diagrams in balanced situations, that is, when a consistent outcome happens at every step. They will be challenged by counting problems where an item is repeated. This seemingly little twist in the problem requires students to count the outcomes differently and makes the problem harder to solve. For example, how many ways can you arrange the letters in the word "FREE." Although you have a total of four letters in the word, there are only three possible choices for the first letter (F, R, or E); the repeated letter E throws a different twist into the construction of the tree diagram, namely it makes it "unbalanced." Look at the tree diagram below. Can you find where a different number of options are possible? Students should notice that after the first choice of a letter "F," there will only be two possible letters that could come next – namely, either R or E. But if their choice for a first letter was "E," they would have three possible letters for their second choice, namely, F, R, or E. When students look at the three subgroups in this tree, they will notice that the structure of the "E" subgroup is different from the structure of "R" subgroup, and from the structure of the "F" subgroup. The tree is not balanced. Continued on next page

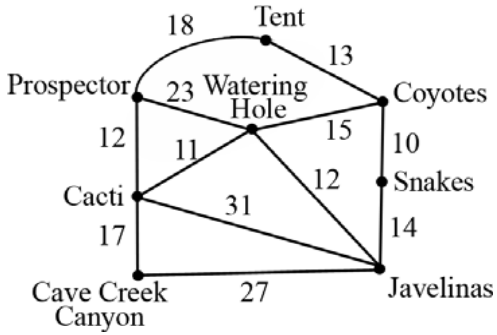
Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
		Example: • All possible arrangements of the letters in the word FREE. 

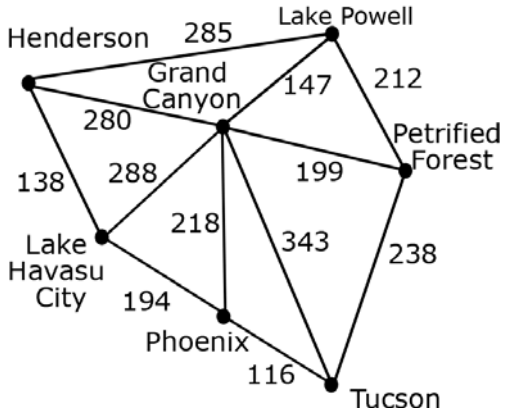
Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 2. Explore counting problems with Venn diagrams using three attributes. Connections: M06-S5C2-07	M06-S5C2-05. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	Example: • Ms. Taft's class has 35 students. Ms. Taft surveyed her students to find out the games they like to play in class. o $\frac{1}{5}$ said they liked to play only dodge ball. o $\frac{2}{5}$ said they like to play only basketball. o $\frac{1}{5}$ said they like to play only soccer. o $\frac{1}{5}$ said they liked to play dodge ball, basketball and soccer. Record the results in a Venn diagram that shows the fraction of students and number of students in each group. What is the total number of students who said they enjoy each sport?

Strand 2: Data Analysis, Probability, and Discrete Mathematics
Concept 4: Vertex-Edge Graphs

Understand and apply vertex-edge graphs.

In Grade 6, students learn about Hamilton paths and circuits in comparison to prior learning of Euler paths and circuits in fifth grade. They learn to solve real-world problems related to Hamilton paths and circuits.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Investigate properties of vertex-edge graphs • Hamilton paths, • Hamilton circuits, and • shortest route. Connections: M06-S2C4-02	M06-S5C2-05. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	A Hamilton path in a vertex-edge graph is a path that starts at some vertex in the graph and visits every other vertex of the graph exactly once. Edges along this path may be repeated. A Hamilton circuit is a Hamilton path that ends at the starting vertex. The shortest route may or may not be a Hamilton path. Depending upon the constraints of a problem, each vertex may not need to be visited. Example • If the park ranger is required to visit every location on the vertex-edge graph below, what route should he take? Where should he begin and end his trip?  Continued on next page

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
		• One possible Hamilton path is: Prospector-Tent-Coyotes-Snakes-Javelinas-Watering Hole-Cacti-Cave Creek Canyon. Can you find other Hamilton paths? • Is it possible to start at one vertex (site) on the vertex-edge graph and visit every other vertex just once and return to the starting vertex? If it is possible, name that circuit. • What is the shortest route between Cave Creek Canyon and the Tent?
PO 2. Solve problems related to Hamilton paths and circuits. Connections: M06-S2C4-01	M06-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Example: • The Clark family is vacationing in the southwestern part of the United States. They are going to visit every location on the graph below. What is the shortest route they can take? Where should the first vacation stop be for the Clark family? The last stop? 

Strand 3: Patterns, Algebra, and Functions

Patterns occur everywhere in nature. Algebraic methods are used to explore, model and describe patterns, relationships, and functions involving numbers, shapes, iteration, recursion, and graphs within a variety of real-world problem solving situations. Iteration and recursion are used to model sequential, step-by-step change. Algebra emphasizes relationships among quantities, including functions, ways of representing mathematical relationships, and the analysis of change.

Concept 1: Patterns

Identify patterns and apply pattern recognition to reason mathematically while integrating content from each of the other strands.

In Grade 6, students expand prior knowledge about sequences involving whole numbers, fractions, and decimals to include sequences that use the four basic operations.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Recognize, describe, create, and analyze a numerical sequence involving fractions and decimals using all four basic operations. Connections: M06-S1C2-02, M06-S1C2-03, M06-S1C2-04, M06-S3C2-01	M06-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Example: • Analyze each of the following sequences. What would the next term be? How did you determine what the next term would be? Write a general rule describing each sequence using words or mathematical symbols. o $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ o $0, 2\frac{1}{2}, 5, 7\frac{1}{2}, \ldots$ o 0.3, 0.5, 0.9, 1.7, …

Strand 3: Patterns, Algebra, and Functions
Concept 2: Functions and Relationships

Describe and model functions and their relationships.

In Grade 6, students examine the relationship between two sets of numbers represented by a chart, graph, table, written language, or an expression.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Recognize and describe a relationship between two quantities, given by a chart, table, or graph, using words and expressions. Connections: M06-S3C1-01, M06-S3C3-03, M06-S3C4-01, SC06-S1C3-01, SC06-S1C3-04, SS06-S2C1-01, SS06-S2C1-02, SS06-S4C1-02	M06-S5C2-03. Analyze and compare mathematical strategies for efficient problem solving; select and use one or more strategies to solve a problem.	Example: • What is the relationship between the two variables? Write an expression that illustrates the relationship. *x* | 1 | 2 | 3 | 4 *y* | 2.5 | 5 | 7.5 | 10

Strand 3: Patterns, Algebra, and Functions
Concept 3: Algebraic Representations

Represent and analyze mathematical situations and structures using algebraic representations.

In Grade 6, students write and use algebraic expressions and equations containing fractions and decimals to represent and solve contextual problems. They extend this skill to create and solve two-step equations containing positive rational coefficients. They use mathematical terminology and symbols to translate between written and verbal mathematical expressions and equations that have the four basic operations.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Use an algebraic expression to represent a quantity in a given context. Connections: M06-S3C3-02, M06-S4C1-02	M06-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	Examples: • Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has. (Solution: 2*c*+3 where *c* represents the number of crayons that Elizabeth has.) • An amusement park charges $28 to enter and $0.35 per ticket. Write an algebraic expression to represent the total amount spent. (Solution: 28 + 0.35*t* where *t* represents the number of tickets purchased.)
PO 2. Create and solve two-step equations that can be solved using inverse properties with fractions and decimals. Connections: M06-S1C3-02, M06-S3C3-01, M06-S4C1-02	M06-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	Students are expected to create and solve two-step equations in which the leading coefficients have positive values. Example: • $\frac{1}{2}n + 7 = 14$

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 3. Translate both ways between a verbal description and an algebraic expression or equation. Connections: M06-S3C2-01, M06-S3C3-01	M06-S5C2-05. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	Examples: • Andrew has a summer job doing yard work. He is paid $15 per hour and a $20 bonus when he completes the yard. He was paid $85 for completing one yard. Write an equation to represent the amount of money he earned. • Describe a problem situation that can be solved using the equation $2C + 3 = 15$; where C represents the cost of an item
PO 4. Evaluate an expression involving the four basic operations by substituting given fractions and decimals for the variable. Connections: M06-S1C2-02, M06-S1C2-03, M06-S1C2-04, M06-S1C3-02, M06-S4C4-04, M06-S4C4-05	M06-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	Example: • $5(n + 3) – 7n$, when $n = \frac{1}{2}$.

Strand 3: Patterns, Algebra, and Functions
Concept 4: Analysis of Change

Analyze how changing the values of one quantity corresponds to change in the values of another quantity.

In Grade 6, students extend prior learning about patterns of change to predict missing values on line graphs or scatterplots.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Determine a pattern to predict missing values on a line graph or scatterplot. Connections: M06-S1C3-02, M06-S3C2-01, SC06-S1C3-01	M06-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Example: • Use the graph below to determine how much money a person makes after working exactly 9 hours. 

Strand 4: Geometry and Measurement

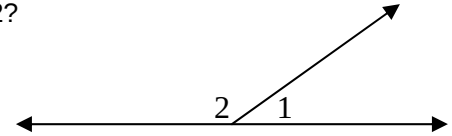
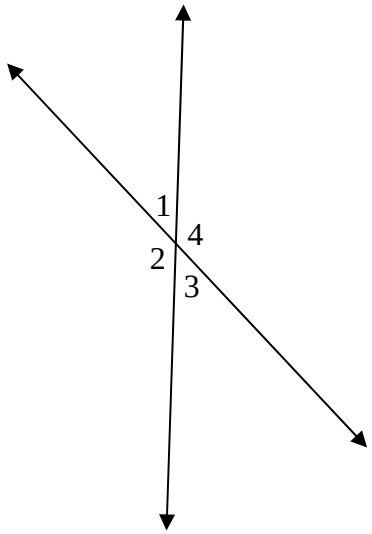
Geometry is a natural place for the development of students' reasoning, higher thinking, and justification skills culminating in work with proofs. Geometric modeling and spatial reasoning offer ways to interpret and describe physical environments and can be important tools in problem solving. Students use geometric methods, properties and relationships, transformations, and coordinate geometry as a means to recognize, draw, describe, connect, analyze, and measure shapes and representations in the physical world. Measurement is the assignment of a numerical value to an attribute of an object, such as the length of a pencil. At more sophisticated levels, measurement involves assigning a number to a characteristic of a situation, as is done by the consumer price index. A major emphasis in this strand is becoming familiar with the units and processes that are used in measuring attributes.

Concept 1: Geometric Properties

Analyze the attributes and properties of 2- and 3- dimensional figures and develop mathematical arguments about their relationships.

In Grade 6, students extend their exploration of 2-dimensional figures to include circles. They investigate the relationship between the radius, diameter and circumference of a circle to define π. Students investigate and solve problems with angle relationships by applying the properties of supplementary, complementary, and vertical angles.

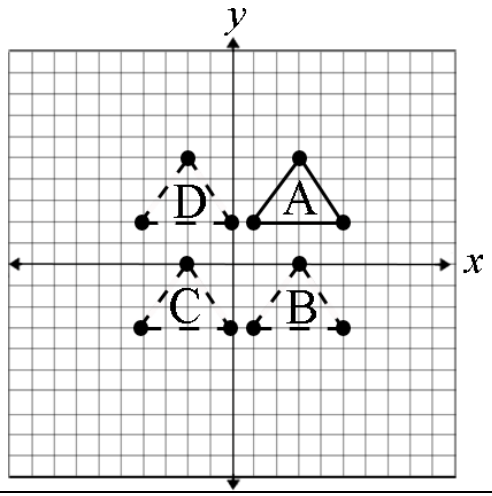
Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Define π (pi) as the ratio between the circumference and diameter of a circle and explain the relationship among the diameter, radius, and circumference.	M06-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Students develop the relationship between the circumference and the diameter, and the circumference and the radius of a circle. The relationships are connected since the diameter is equal to two radii. Example: • Measure the diameter and circumference of three circular objects in the classroom. Add your measurements to the class data chart and graph. Describe the pattern that you see in the data. Write the table in terms of the radius versus the circumference. Describe the pattern that you see in the data. Write a paragraph about the relationship between the diameter, radius, and circumference of a circle.

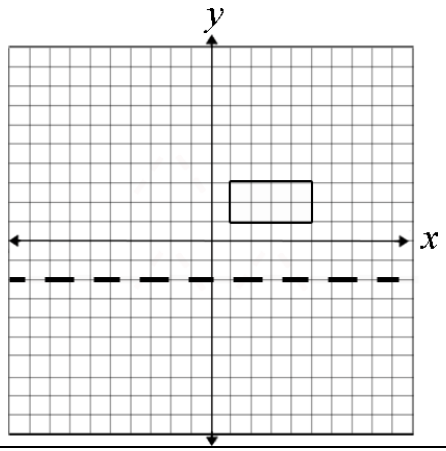
Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 2. Solve problems using properties of supplementary, complementary, and vertical angles. Connections: M06-S3C3-01, M06-S3C3-02	M06-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Examples: • If the measure of $\angle 1 = 35^\circ$, what is the measure of $\angle 2$?  • If the measure of $\angle 2 = 135^\circ$, what are the measures of all of the other angles? Explain the properties that you used to figure out the measures. 

Strand 4: Geometry and Measurement
Concept 2: Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

In Grade 6, students build on their knowledge of translations and reflections to perform transformations in all four quadrants of the coordinate plane. They differentiate between vertical and horizontal lines of reflection to reflect polygons in all four quadrants.

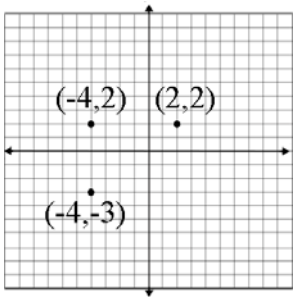
Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Identify a simple translation or reflection and model its effect on a 2-dimensional figure on a coordinate plane using all four quadrants. Connections: M06-S4C2-02	M06-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Example: • Triangle A is in quadrant I. It is moved five units to the left and five units down. Which triangle below shows this transformation? 

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 2. Draw a reflection of a polygon in the coordinate plane using a horizontal or vertical line of reflection. Connections: M06-S4C2-01, M06-S4C3-01, M06-S4C3-02	M06-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Example: • Draw the reflection of the rectangle using the dotted line as the line of reflection. 

Strand 4: Geometry and Measurement
Concept 3: Coordinate Geometry

Specify and describe spatial relationships using rectangular and other coordinate systems while integrating content from each of the other strands.

In Grade 6, students expand their understanding of graphing ordered pairs to all four quadrants. They use their understanding of geometric properties to justify the location of a missing coordinate in a figure.

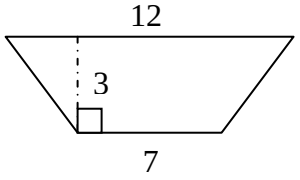
Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Graph ordered pairs in any quadrant of the coordinate plane. Connections: M06-S4C2-02, M06-S4C3-02		Example: • Graph and label the points below on a coordinate plane. o A (0, 0) o B (2, -4) o C (5, 5) o D (-4, 1) o E (2.5, -6) o F (-3, -2)
PO 2. State the missing coordinate of a given figure on the coordinate plane using geometric properties to justify the solution. Connections: M06-S4C2-02, M06-S4C3-01	M06-S5C2-04. Apply a previously used problem-solving strategy in a new context.	Example: • If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? 

Strand 4: Geometry and Measurement
Concept 4: Measurement

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

In Grade 6, students build upon their prior knowledge of measurement to determine the appropriate unit of measure, tool, and necessary precision to solve problems. They convert within systems of measurement to solve problems. They use scale drawings to estimate the measure of an object. Students also apply formulas for area and perimeter to solve problems and explore the relationship between volume and area.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Determine the appropriate unit of measure for a given context and the appropriate tool to measure to the needed precision (including length, capacity, angles, time, and mass). Connections: M06-S1C3-02, SC06-S1C2-04	M06-S5C2-01. Analyze a problem situation to determine the question(s) to be answered.	Example: • In your science class, you want to measure leaf width and plant heights to determine the effects of different kinds of fertilizers. What tools and units of measures would you use to make the measurements? To what degree of precision should you measure? Explain and justify your choices.
PO 2. Solve problems involving conversion within the U.S. Customary and within the metric system. Connections: M06-S1C1-03, M06-S1C3-02	M06-S5C2-04. Apply a previously used problem-solving strategy in a new context.	
PO 3. Estimate the measure of objects using a scale drawing or map. Connections: M06-S1C1-03, M06-S1C3-02, SS06-S4C1-03	M06-S5C2-03. Analyze and compare mathematical strategies for efficient problem solving; select and use one or more strategies to solve a problem.	Example: • On a drawing of an airplane, 2.5 inches is the same as 10 feet on an actual airplane. Estimate the length of the actual plane if the scale drawing shows a length of 5.75 inches.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 4. Solve problems involving the area of simple polygons using formulas for rectangles and triangles. Connections: M06-S1C3-02, M06-S3C3-04, M06-S5C1-02	M06-S5C2-02. Identify relevant, missing, and extraneous information related to the solution to a problem. M06-S5C2-04. Apply a previously used problem-solving strategy in a new context.	Examples: • Find the area of a triangle with a base length of three units and a height of four units. • Find the area of the trapezoid shown below using the formulas for rectangles and triangles. 
PO 5. Solve problems involving area and perimeter of regular and irregular polygons. Connections: M06-S1C3-02, M06-S3C3-04, M06-S5C1-02	M06-S5C2-04. Apply a previously used problem-solving strategy in a new context.	Examples: • A rectangle measures 3 inches by 4 inches. If the lengths of each side double, what is the effect on the area? What is the effect on the perimeter? • The area of the rectangular school garden is 24 square units. The length of the garden is 8 units. What is the length of the fence needed to enclose the entire garden?
PO 6. Describe the relationship between the volume of a figure and the area of its base.	M06-S5C2-04. Apply a previously used problem-solving strategy in a new context.	Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Students derive the volume formula (volume equals the area of the base times the height) and explore how this idea would apply to other prisms and cylinders.

Strand 5: Structure and Logic
This strand emphasizes the core processes of problem solving. Students draw from the content of the other four strands to devise algorithms and analyze algorithmic thinking. Strand One and Strand Three provide the conceptual and computational basis for these algorithms. Logical reasoning and proof draws its substance from the study of geometry, patterns, and analysis to connect remaining strands. Students use algorithms, algorithmic thinking, and logical reasoning (both inductive and deductive) as they make conjectures and test the validity of arguments and proofs. Concept two develops the core processes as students evaluate situations, select problem solving strategies, draw logical conclusions, develop and describe solutions, and recognize their applications.

Concept 1: Algorithms and Algorithmic Thinking

Use reasoning to solve mathematical problems.

In Grade 6, students expand their understanding of algorithms to analyzing algorithms for multiplying and dividing fractions and decimals using properties of the real number system. They use their knowledge of parallelograms and triangles to create and defend algorithms for calculating the area of compound figures.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Analyze algorithms for multiplying and dividing fractions and decimals using the associative, commutative, and distributive properties Connections: M06-S1C2-02, M06-S1C2-03, M06-S1C2-04, M06-S1C2-05	M06-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Examples: • Commutative Property 7 • 0.359 becomes 0.359 • 7 to set up the multiplication problem with the most number of digits above the number with the least number of digits. • Associative Property 0.47 • 7.3 • 1.8 can be written as (0.47 • 7.3) • 1.8 to allow the product of the first two numbers to be multiplied by the third number. • Distributive Property 7 • 5 ½ can be written as 7 (5 + ½) and then distributed to get 7 • 5 + 7 • ½

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 2. Create and justify an algorithm to determine the area of a given compound figure using parallelograms and triangles. Connections: M06S4C4-04, M06S4C4-05	M06-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Justifications may include numbers, words, a model of physical objects, or equations.

Strand 5: Structure and Logic
Concept 2: Logic, Reasoning, Problem Solving, and Proof

Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize their applications.

In Grade 6, students continue to use a variety of problem-solving strategies, and analyze them for efficiency and appropriateness for contextual situations. They communicate their thinking using multiple representations, synthesize and organize information from multiple sources to make inferences, draw conclusions, and justify their reasoning. Students begin to solve simple logic problems using conditional statements.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:	Some of the Strand 5 Concept 2 performance objectives are listed throughout the grade level document in the Process Integration Column (2nd column). Since these performance objectives are connected to the other content strands, the process integration column is not used in this section next to those performance objectives.	
PO 1. Analyze a problem situation to determine the question(s) to be answered. Connections: SC06-S1C1-02		
PO 2. Identify relevant, missing, and extraneous information related to the solution to a problem.		
PO 3. Analyze and compare mathematical strategies for efficient problem solving; select and use one or more strategies to solve a problem.		Students are expected to determine what information is needed to solve a problems and if the problem cannot be solved, which information is missing. If possible, students should state their assumption about the missing information and solve the problem using their assumptions.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:	Some of the Strand 5 Concept 2 performance objectives are listed throughout the grade level document in the Process Integration Column (2nd column). Since these performance objectives are connected to the other content strands, the process integration column is not used in this section next to those performance objectives.	
PO 4. Apply a previously used problem-solving strategy in a new context.		Multiple representations may include but are not limited to numbers, symbols, graphs, equations, pictures, or words.
PO 5. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution. Connections: SC06-S1C4-02		
PO 6. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language. Connections: SC06-S1C4-03		Students are expected to begin to use formal notation in expressing algebraic and geometric concepts.
PO 7. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning. Connections: M06-S2C3-02, SC06-S1C3-02, SS06-S1C1-07, SS06-S2C1-07, SS06-S4C4-03		Students need multiple opportunities to make inferences, draw conclusions and justify their reasoning using problems from all of the content strands. Students are expected to write justifications and explain their thinking to other students.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:	Some of the Strand 5 Concept 2 performance objectives are listed throughout the grade level document in the Process Integration Column (2nd column). Since these performance objectives are connected to the other content strands, the process integration column is not used in this section next to those performance objectives.	
PO 8. Make and test conjectures based on information collected from explorations and experiments.		
PO 9. Solve simple logic problems, including conditional statements, and justify solution methods and reasoning.	M07-S5C2-03. Analyze and compare mathematical strategies for efficient problem solving; select and use one or more strategies to solve a problem. M06-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Example: • In a magic square below, if the sum of every row and column is the same, then what values can be placed in the empty boxes? Explain how you know your answer is correct. 6 | 7 | 8 | 3 | 4